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ENGS 26 – Control Theory

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Inverted Pendulum Car

1 Modeling

1.1 Built-in amp

The power amp in the vehicle allows for the small current output from the op-amps to drive the motor at a certain gain. Therefore, its transfer function is:

$$G_{amp} = K_a$$

1.2 Motor

The motor is a simple component that we have worked with all term. It has one pole at 1/***T*** and a gain factor K_m. Therefore, its transfer function is:

$$G_m = \frac{K_m}{Ts + 1}$$

1.3 Gear Ratio

The vehicle uses a gear ratio to compensate for the low torque of the electric motor. When looking at angular momentum it can be treated as a gain related to the ratio of the radii of the input (R_1) and output (R_2) shafts. The transfer function is as follows:

$$\mathrm{GR(s)} = \frac{R_1}{R_2}$$

1.4 Angular velocity to forces

A relationship between the the input angular velocity (***w***) from the motor to an output force (***u***) on the car can be established through the motor torque. The angular momentum of the wheels (***J***) and the radius of the wheel (***R_w***) must be taken into account, and the physical model is described as the following:

$$u * R_w = J * \dot{\omega}$$

And the following transfer function is established:

$$G_{conv}(s) = \frac{J * s}{R_w}$$

1.5 Inverted Pendulum and Car

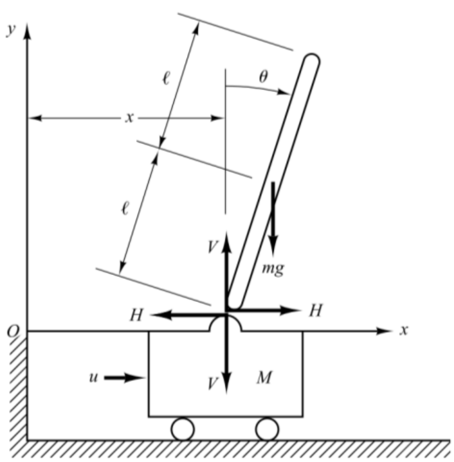

Figure 1 Inverted Pendulum Car (Ogata)

The inverted pendulum car was modeled from an input force (***u***) to the output angle of the stick (θ). The length of the stick to its center of mass is labeled as ***l***, and the mass is ***m***, and the moment of inertia is ***I***. The mass of the car itself is labeled as ***M***. The vertical and horizontal forces for the car on the stick and vice versa are labeled as ***V*** and ***H***, respectively. The gravitational constant (***g***) is 9.8 m/s^2.

The motion of the car can be characterized as follows:

$$M\ddot{x} = -H + u$$

Note that the motion of the car in the vertical direction can be ignored, as the car is not expected to launch. The motion of the stick can be split up as the linear motion and its angular motion as follows:

$$m\ddot{x} = m\frac{d^2}{dt^2}(x + l\sin\theta) = H$$
$$m\frac{d^2}{dt^2}(l\cos\theta) = V - mg$$
$$I\ddot{\theta} = V * l\sin\theta - H * l\cos\theta$$

Through linearization and the use of a few relationships of when $\boldsymbol{\theta}$ is small, we arrive at the following:

$$(M + m)\ddot{x} + ml\ddot{\theta} = u$$
$$(I + ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$$

Using this mathematical model of the system the transfer function can be derived as shown:

$$G_{pend}(s) = \frac{-ml}{[(I + ml^2)(M + m) - m^2l^2]s^2 - (M + m)mgl}$$

1.6 Sensor

The sensor outputs a voltage based on the angle of the stick and is considered relatively linear to the angle around the center point. The data collected from the lab is shown in the graph below. The sensor is then modeled as a gain and the offset can treated as a disturbance, which is compensated for by setting V_{ref} equal to it. The transfer function is as shown:

$$G_{sensor}(s) = K_s$$

1.7 Open Loop Plant Transfer Function

$$G(s) = G_{pend}(s) * G_{sensor}(s) * G_{amp}(s) * G_m(s) * G_{conv}(s) * GR(s)$$

2 Analytical Design

The car we used for the project was PD-10. It is a newer car that has a more streamlined circuit design.

2.1 Car Components and Values

Our car had three switches: one for the mains, one for the motor, and one for the sensor (See figure 2). It also only had one set of batteries that provided all the power.

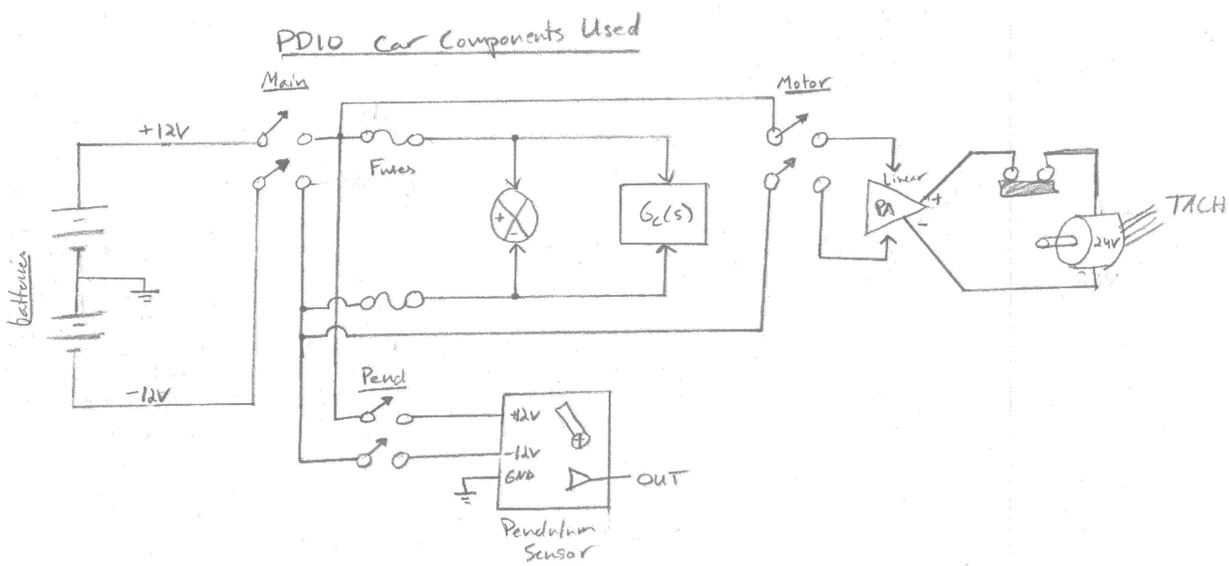

Figure 2 PD10 Car Component Diagram

We then tested each of the components to determine their values for the MATLAB model. We followed the procedures in lab 3 to determine motor to have values of ***Km*** to be 13.0 rad/(V*sec) and ***T*** to be 62.5ms. We followed the lab 5 procedure to find the sensor has a gain value of 571.89V/rad. The power amp has a gain of 1.25. The gear ratio was 0.25.

2.2 Circuit Design from MATLAB

We used MATLAB to calculate the moments of inertia and the open loop transfer function of:

$$G(s) = \frac{0.003661\, s}{1.229e-05\, s^3 + 0.0001966\, s^2 - 0.0001855\, s - 0.002967}$$

We then used PID Tuner to calculate the analytical values to a disturbance (See figure 3). Our gains are pretty small which makes sense given that we cannot go over 12v.

Controller Parameters

	Tuned
Kp	0.18775
Ki	1.1601
Kd	0.0075959
Tf	n/a

Figure 3 MATLAB Derived PID Parameters

2.3 *Expected Performance*

Based on the MATLAB data our car should have a disturbance rejection response like figure X. It should oscillate back and forth at first with a max overshoot of 24.2%. Then it will approach steady state and calm down within 3.2 seconds.

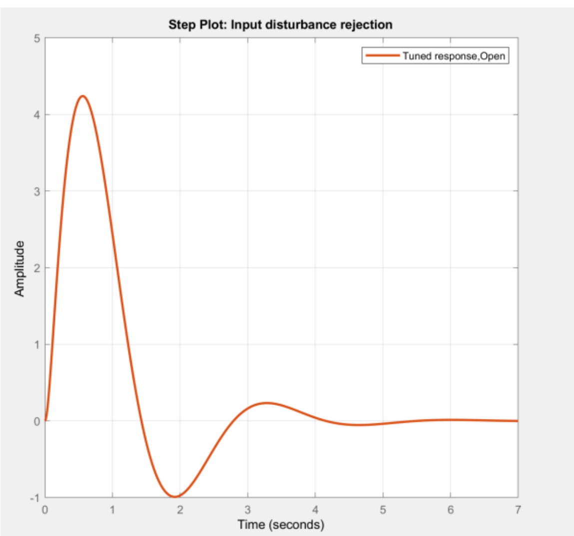

Figure 4 MATLAB Distrubance Plot

Performance and Robustness

	Tuned
Rise time	0.563 seconds
Settling time	3.2 seconds
Overshoot	24.2 %
Peak	4.12
Gain margin	-Inf dB @ 0 rad/s
Phase margin	15 deg @ 2.69 rad/s
Closed-loop stability	Stable

Figure 5 MATLAB Disturbance Plot Analysis

3 Implementation

3.1 *Actual Components*

For our actual PID controller, we designed the distinct parts in parallel, so it was easier to tune and modify. This way we could better see the effect each section had on the system. In addition, we had a summing junction that combined a zero-reference voltage with the negative feedback voltage. We also needed an inverter for the PID section due to how the op-amps function. The gains for the PID controller were as follows: $\mathbf{K_i}$ is .4340, the $\mathbf{K_p}$ is .4557, and the $\mathbf{K_d}$ is .0095. Figure 6 shows the schematics of our controller and Figure 7 shows our actual implementation. Going from left to right the op-amps are: summing junction, $\mathbf{K_p}$ gain, $\mathbf{K_d}$ gain, $\mathbf{K_I}$ gain, inverter.

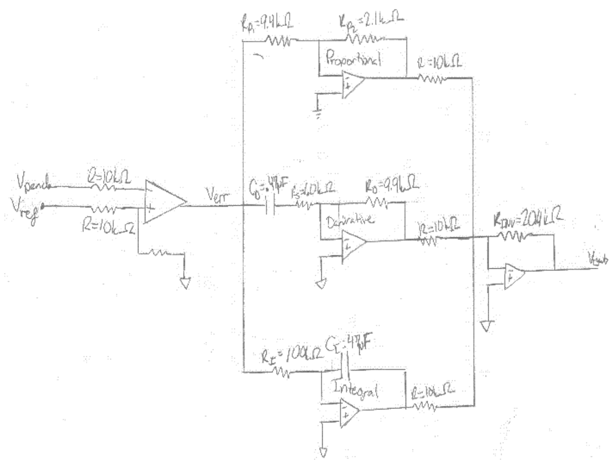

Figure 6 Op-Amp Circuit Design Diagram

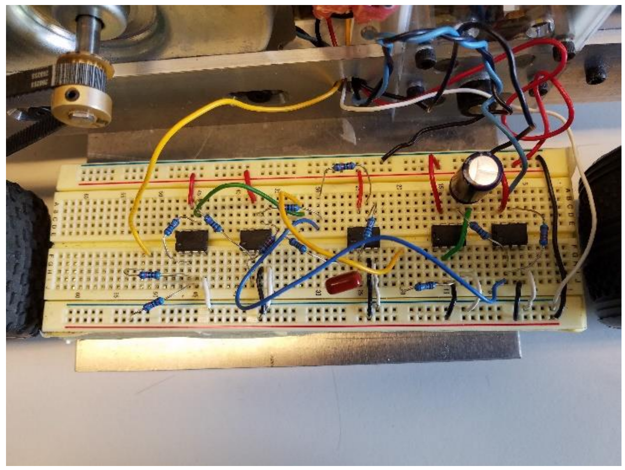
Figure 7 Photo of Circuit on Car

3.2 Actual Results

For our actual car, when given a hardy tap it had a settling time of 12.2 seconds. It never quite stopped still but that is more because of the delicacy of the system rather than our implementation. However, we did succeed in having the car move very smoothly and slowly with a predictable oscillation in steady state. We also lowered the gain to make sure there was no slipping of the wheels and that we were not putting more than 12v into the motor.

4 Discussion

4.1 Error

The main error was in the transient response. The MATLAB treated it as a perfect system and ours could not perform as accurately. The difference was 9.0 seconds. In addition, we could never get the car to perfectly stop, but that would be caused by air flow and how extremely delicate the system is to outside disturbances.

The other thing that came up in our testing was that the car performed better with a zero-reference voltage than with what the sensor testing predicted. This makes sense for the transient response because having the reference voltage lowers the maximum error in one direction but raises it in the other direction. This makes the car a little more unpredictable and difficult to model symmetrically. Furthermore, we hypothesis that having a slight offset with the error signal at steady state makes the car movement more predictable; whereas if we truly had a zero error the rod with fall in either direction randomly and cause the car to be more jittery.

4.2 Improvements

The main problem with our MATLAB design is that it did not limit the max overshoot the same way the box does for the real car. MATLAB would therefore allow the system to become more unstable than it does in real life. The saturation that the car undergoes means that gains can be a lot lower and still produce desirable results. For example, if given a hard enough tap the pendulum would hit the side wall of the sensor box.

The other main problem we faced was the lack of ideal movement response. This was primarily caused by wheel slippage since we did not have a way to model the limits friction puts on the wheel acceleration. Additionally, we had some trouble with high frequency motor vibration that was not translating to real motion due to slack in the motor belt, in addition to longer transient response.